

**Title:** M A T H E M A T I C S  
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**Brief Overview:**

This lesson will demonstrate a current technical trade application (computer numerical control and using machine shop formulas). Students will use the coordinate plane and trigonometric functions to mill a piece of metal. They will find the distance between points using trigonometry, reproduce an xy table, and use TI-82 graphing calculators and the CBL to sketch a similar form.

**Link to Standards:**

- **Problem Solving** Students will demonstrate their ability to solve "real-shop" mathematical problems with the use of technology.
- **Communications** Students will be able to demonstrate their understanding by discussing the results of their findings
- **Reasoning** Students will demonstrate their ability to reason indirectly and deductively by making conjectures based upon their findings.
- **Connections** Students will connect trigonometry and algebra with a real-life machine shop procedure.
- **Algebra** Students will demonstrate their ability to apply algebraic concepts, Cartesian coordinate graphing as well as solving linear equations.
- **Statistics** Students will be able to select substantial data and eliminate extraneous data.

**Grade/Level:**

Grades 9–12; Trigonometry/Pre-Calculus/Algebra II

**Duration/Length:**

This lesson will take 3 or 4 periods (45 min.) or two 90-minute periods.

**Prerequisite Knowledge:**

Students should have working knowledge of the following:

- Solving right triangles
- Making x-y tables, and graphing in two dimensions
- Using trigonometric functions
- Using a graphing calculator (TI-82/TI-83)

**Objectives:**

Students will be able to:

- understand the unique input of the Cartesian coordinate system in identifying any reference point in relation to coordinate dimension reference point in relation to coordinate dimensions that are perpendicular to the x and y axes.
- diagram, label, and interpret relationships of the angles and sides of right triangles.
- work cooperatively in groups.
- collect and organize data from resources.
- represent and use numbers in a variety of equivalent forms.
- evaluate a situation and give appropriate support for their answer.

**Materials/Resources/Printed Materials:**

- 30 TI-82 or TI-83 graphing calculators
- 30 CBL units
- 15 meter sticks
- 30 Vernier CBL Motion Detectors
- Tape
- 15 Dowel rods, about 1" thick
- 15 copies of Student Exercise 1 and Student Example
- 4 Teacher Resource Sheets
- Teacher Instructions sheet

**Development/Procedures:**

The teacher will demonstrate one example of how to calculate unknown distances given some distances, angles, and a polygon. Also, the teacher will show how to set up the milling of a piece of metal like the diagram with absolute dimensioning and with a computer numerical control system using appropriate formulas. Then the teacher will demonstrate how to use the CBL and TI-82 to sketch a similar form. Refer to Teacher Instructions sheet and Teacher Resource Sheets to confirm results.

The class will be divided in groups. Each group will receive student example 1, 2 TI-82s, 2 CBLs, 2 Vernier CBL motion detectors, tape, and 1 dowel rod. Each group must work cooperatively to reach a conclusion on a similar example called student exercise 1.

**Evaluation:**

The teacher will circulate around the room to ensure teams are on task and will answer questions students may have. Evaluation will be based upon group performance, computer printed sketch, student exercise 1 calculations, oral presentation (which will include an opportunity for students to make a conclusion and also cite an application of their experimental results) and "evaluating mill," a unit evaluation sheet.

**Extension/Follow Up:**

In drilling, three dimensions need to be used and the middle of the piece may be used as the starting point for absolute dimensioning. In the teacher's resource section a problem of this type is given.

**Authors:**

Pete Anderson  
Mergenthaler High  
Baltimore City, MD

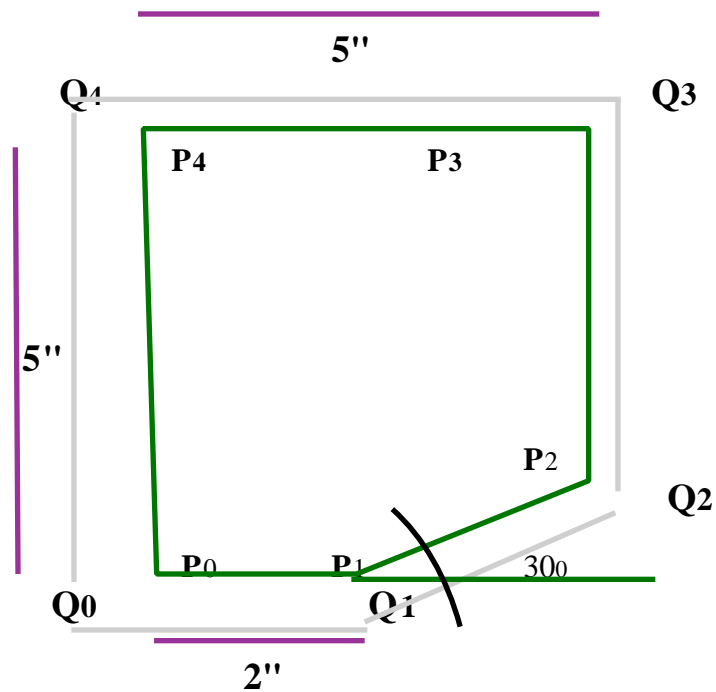
Maria Waltemeyer  
Patterson High School  
Baltimore City, MD

## Calculation of Auxiliary Points

### Example 1

You program the path of the milling axis Qo/Q1/Q2/Q3.....

Milling cutter diameter is .500"



Calculate the x and y-coordinates of all P points on the student recording sheet for the example (next page).

## Student recording sheet for example

		X	Y	Feedrate	Trig. Calculations
<b>Rapid move</b>	G00			exact numbers are not given but see example below	
	G01				
	G01				
<b>Straight Line Move</b>	G01				
	G01				
	G01				
<b>Ending</b>	M05				
<b>Turnoff</b>	M30				

### FORMULAS

$RPM = (4 * \text{cutting speed}) / \text{diameter}$

Read cutting speed from chart such as

Aluminum 2 tooth cutter feed per tooth 1/4 in. endmill .003 in. SFPM Finishing 800

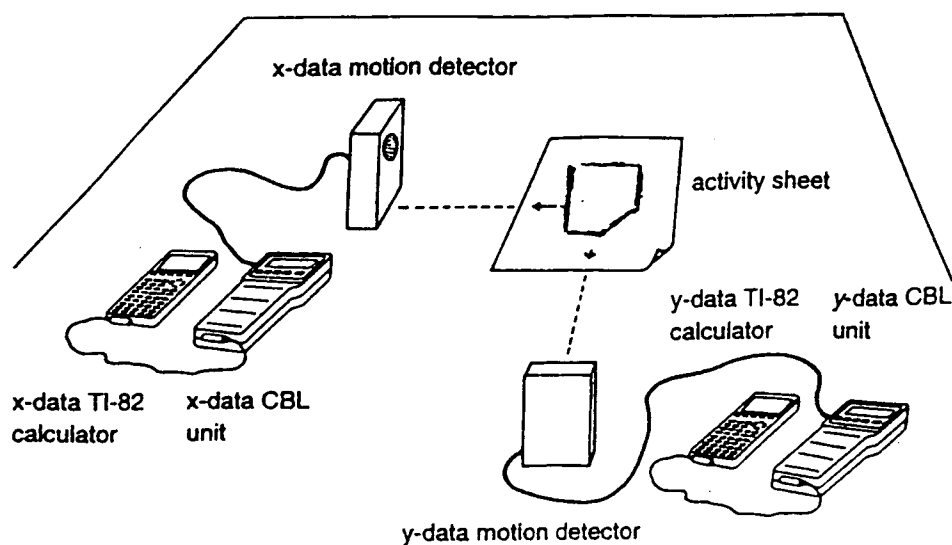
Thus, in example,  $RPM = (4 * 800) / .500 = 6400$

$FEEDRATE = RPM * TOOTH * TOOTHRATE$

In example,  $FEEDRATE = 6400 * 2 * .003 = 38.4 \text{ in./ min}$

NOTE: in examples with a smaller diameter RPM may be more, but you have to cut down on the feedrate because otherwise the tool might bend.

## Student directions for CBL/TI-82 set up for example and exercise 1



1. Use this figure.
2. Tape the pattern sheet to the table.(You will trace over the pattern with the dowel rod in the course of this exercise.)
3. Set up the calculators and motion detectors as shown in the figure.  
Make careful note of which calculator will collect x-data and which will collect y-data.
4. After the motion detectors have been activated , you will have approximately 10 seconds to trace the pattern.
5. Start the DISTFORM program on each of your TI-82 calculators.
6. Follow the instructions on the TI-82 screen to complete the activity.

### ACTIVITY DATA

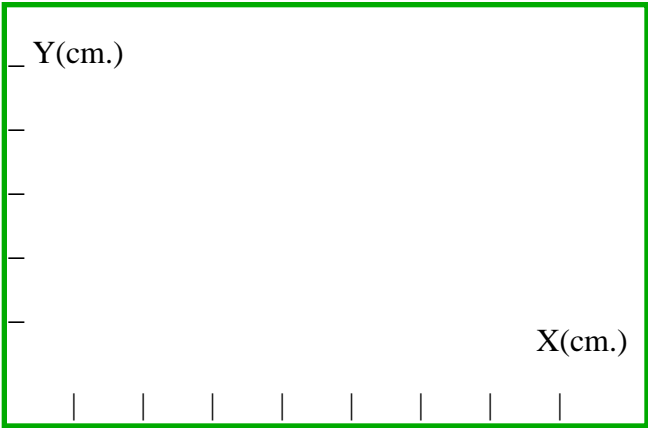
The plot that appears on the TI-82 screen should look like the polygon on the pattern sheet.

- If you are not satisfied with your results, press **CLEAR ENTER** to start again.
- If you are satisfied with your results, press **TRACE** to move the cursor along the plot line. Identify the coordinates of each of the points labeled on the pattern sheet and write them in the table listed below. Give all coordinates in centimeters, rounding to the nearest tenth of a centimeter.
- If your graph is very small, use the Zoom Box option to enlarge it.

**Student recording sheet for CBL/TI-82 example**

POINT	x-coordinate	y-coordinate
P <sub>0</sub>		
P <sub>1</sub>		
P <sub>2</sub>		
P <sub>3</sub>		
P <sub>4</sub>		

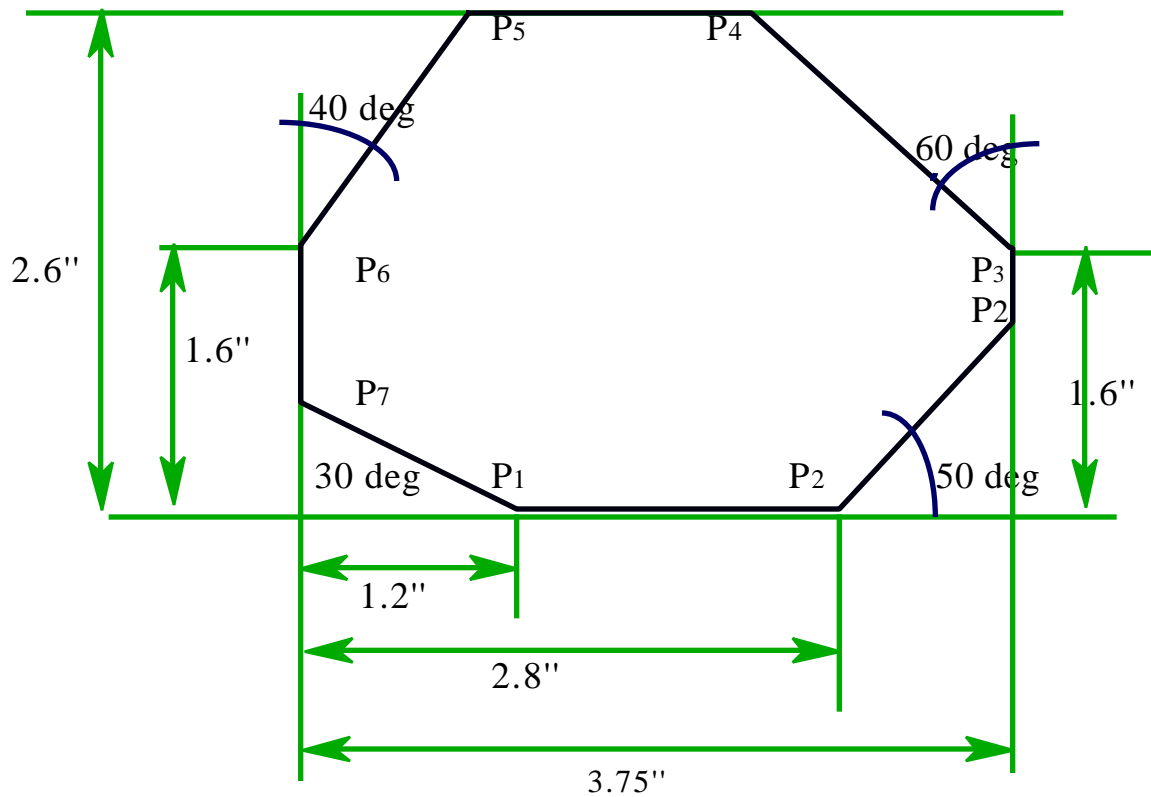
Sketch a picture of your polygon in the space provided below



# Calculation of Auxiliary Points

## Exercise 1

Cutter diameter is .250"



Calculate the x and y-coordinates of all P points and also show trigonometry calculations on the student recording sheet for the exercise (next page).



## Student recording exercise 1 sheet

		X	Y	Feedrate	Trig. Calculations
<b>Rapid move</b>	G00			exact numbers are not given but see example below	
	G01				
	G01				
<b>Straight Line Move</b>	G01				
	G01				
	G01				
	G01				
	G01				
	G01				
	G01				
	G01				
	G01				
	G01				
	G01				
<b>Ending</b>	M05				
<b>Turnoff</b>	M30				

### FORMULAS

$RPM = (4 * \text{cutting speed}) / \text{diameter}$

Read cutting speed from chart such as

Aluminum 2 tooth cutter feed per tooth 1/4 in. endmill .003 in. SFPM Finishing 800

Thus, in example,  $RPM =$

$FEEDRATE = RPM * TOOTH * TOOTHRATE$

In example,  $FEEDRATE =$

NOTE: in examples with a smaller diameter RPM may be more, but you have to cut down on the feedrate because otherwise the tool might bend.

**Teacher Resource Sheet #1**  
**Solution to example**

		X	Y	Feedrate	Trig. Calculations
<b>Rapid move</b>	GOO	<b>0</b>	<b>0</b>	exact numbers are not given but see example below	
	GO1	<b>2</b>	<b>0</b>		
	GO1	<b>5</b>	<b>0</b>		<b>tan 30 deg. = x / 3</b>
<b>Line Move</b>	GO1	<b>5</b>	<b>5</b>		
	GO1	<b>0</b>	<b>5</b>		
	GO1	<b>0</b>	<b>0</b>		
<b>Ending</b>	MO5				
<b>Turnoff</b>	M30				

**FORMULAS**

$RPM = (4 * \text{cutting speed}) / \text{diameter}$

Read cutting speed from chart such as

Aluminum 2 tooth cutter feed per tooth 1/4 in. endmill .003 in. SFPM Finishing 800

Thus, in example,  $RPM = (4 * 800) / .500 = 6400$

$FEEDRATE = RPM * TOOTH * TOOTHRATE$

In example,  $FEEDRATE = 6400 * 2 * .003 = 38.4 \text{ in./ min}$

NOTE: in examples with a smaller diameter RPM may be more, but you have to cut down on the feedrate because otherwise the tool might bend.

**Teacher Resource Sheet #2**  
**Teacher's resource solution to student exercise 1 sheet**

		X	Y	Feedrate	Trig. Calculations
<b>Rapid move</b>	GOO	<b>0</b>	<b>0</b>	exact numbers are not given but see example below	
	GO1	<b>1.6</b>	<b>0</b>		
	GO1	<b>2.55</b>	<b>0</b>		
<b>Straight Line Move</b>	GO1	<b>2.55</b>	<b>1.13</b>		<b>tan 50 deg. = x / .95</b>
	GO1	<b>2.55</b>	<b>1.6</b>		
	GO1	<b>2.55</b>	<b>2.6</b>		
	GO1	<b>.82</b>	<b>2.6</b>		<b>tan 60 deg. = x / 1</b>
	GO1	<b>-.36</b>	<b>2.6</b>		<b>tan 40 deg. = x / 1</b>
	GO1	<b>-1.2</b>	<b>2.6</b>		
	GO1	<b>-1.2</b>	<b>1.6</b>		
	GO1	<b>-1.2</b>	<b>.69</b>		<b>tan 30 deg. = x / 1.2</b>
	GO1	<b>-1.2</b>	<b>0</b>		
	GO1	<b>0</b>	<b>0</b>		
<b>Ending</b>	MO5				
<b>Turnoff</b>	M30				

**FORMULAS**

$RPM = (4 * \text{cutting speed}) / \text{diameter}$

Read cutting speed from chart such as

Aluminum 2 tooth cutter feed per tooth 1/4 in. endmill .003 in. SFPM Finishing 800

Thus, in example,  $RPM = (4 * 800) / .250 = 12800$

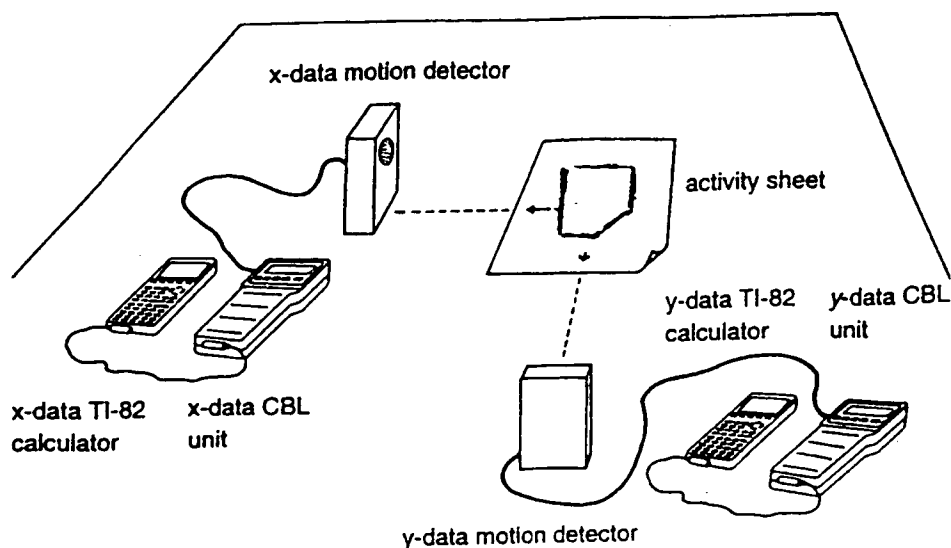
$FEEDRATE = RPM * TOOTH * TOOTHRATE$

In example,  $FEEDRATE = 12800 * 2 * .003 = 76.8 \text{ in./ min}$

NOTE: in examples with a smaller diameter RPM may be more, but you have to cut down on the feedrate because otherwise the tool might bend

### TEACHER's INSTRUCTIONS:

In this activity, the motion of the dowel rod tracing out a certain pattern will be recorded by a pair of motion detectors. It is very important to hold the rod vertically, not tilted to one side or the other. The detectors must be carefully arranged so that nothing obstructs the path between each detector and the dowel rod.



1. Use this figure.
2. Tape the pattern sheet to the table. (You will trace over the pattern with the dowel rod in the course of this exercise.)
3. Set up the calculators and motion detectors as shown in the figure. Make careful note of which calculator will collect x-data and which will collect y-data.
4. After the motion detectors have been activated, you will have approximately 10 seconds to trace the pattern.
5. Start the DISTFORM program on each of your TI-82 calculators.
6. Follow the instructions on the TI-82 screen to complete the activity.

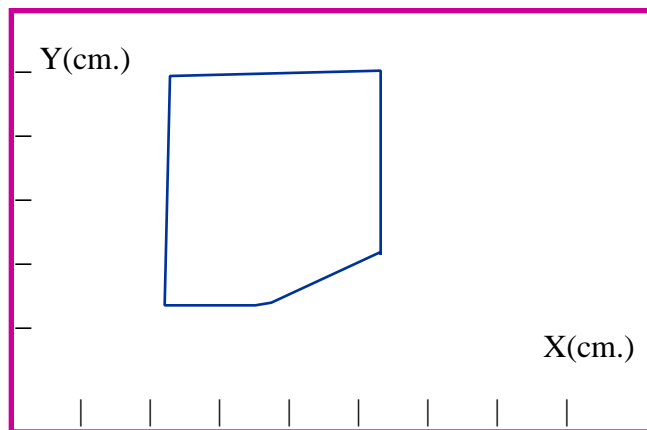
### ACTIVITY DATA

The plot that appears on the TI-82 screen should look like the polygon on the pattern sheet.

- If you are not satisfied with your results, press **CLEAR ENTER** to start again.
- If you are satisfied with your results, press **TRACE** to move the cursor along the plot line. Identify the coordinates of each of the points labeled on the pattern sheet and write them in the table listed below. Give all coordinates in centimeters, rounding to the nearest tenth of a centimeter.
- If your graph is very small, use the Zoom Box option to enlarge it.

**Teacher Resource Sheet #3****Teacher's answers for CBL/TI-82 example ( all numbers are a rough estimate)**

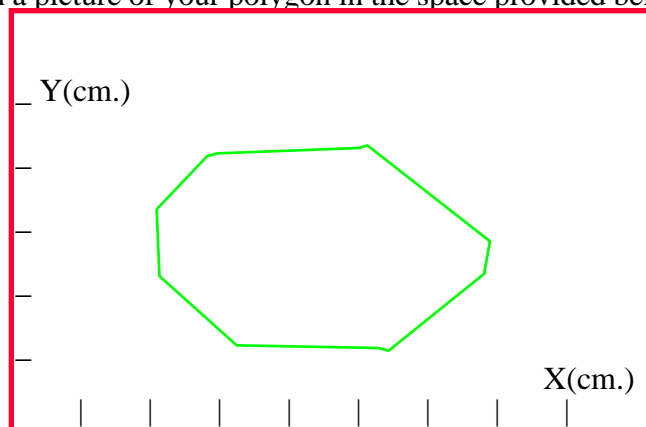
POINT	x-coordinate	y-coordinate
P <sub>0</sub>	63.34	59.93
P <sub>1</sub>	66.23	60.23
P <sub>2</sub>	71.23	61.47
P <sub>3</sub>	70.25	68.93
P <sub>4</sub>	59.61	68.16

**Sketch a picture of your polygon in the space provided below ( rough estimate something like this)**

**Teacher Resource Sheet #4****Teacher's recording for CBL/TI-82 exercise 1(all numbers are a rough estimate)**

POINT	x-coordinate	y-coordinate
P <sub>0</sub>	58	58
P <sub>1</sub>	62	58
P <sub>2</sub>	78	64
P <sub>3</sub>	78	66
P <sub>4</sub>	78	68
P <sub>5</sub>	61	68
P <sub>6</sub>	57	66
P <sub>7</sub>	57	59

Sketch a picture of your polygon in the space provided below(it will be rough estimate)



## *Evaluating mill*

- 1. Did your group enjoy this unit?*
  
- 2. What did you learn in this unit? (conclusion)*
  
- 3. Which activity in this unit did you enjoy the most? Think of another shop application of your experiment.*
  
- 4. If given the choice, would you eliminate or change anything from this unit?*
  
- 5. Any other comments? Please explain.*

*NAME OF GROUP:*

*DATE:*

*Your name (optional):*

## Extension exercise

### NC Rectangular Coordinate Dimensioning

1. a. Float the reference point to the center of the Mounting Plate.
- b. Determine the x,y, and z coordinate dimensions for holes# 1,2,3, and 4.

Hole	X	Y	Z
	(0.001")		
<b>1</b>			
<b>2</b>			
<b>3</b>			
<b>4</b>			

